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STARS OF BOSONS WITH NON-MINIMAL ENERGY-MOMENTUM TENSOR

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ABSTRACT

We obtain spherically symmetric solutions for scalar fields with a non-minimal coupling $\xi \mid \phi \mid^2 R$ to gravity. We find, for fields of mass m, maximum masses and number of particles of order $M_{max} \sim 0.73\xi^{1/2}\ M^2_{Planck}/m$, and $N_{max} \sim 0.88\xi^{1/2}\ M^2_{Planck}/m^2$ respectively, for large positive ξ. For large negative ξ we find, $M_{max} \sim 0.66 \mid \xi \mid^{1/2}\ M^2_{Planck}/m$,and $N_{max} \sim 0.72 \mid \xi \mid^{1/2} M^2_{Planck}/m^2$.

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The importance that is presently being attributed to different sorts of exotic scalar fields to the dynamics of the early Universe has renewed the interest in studying the possibility that, due to a gravitational condensation mechanism, these fields could form spherically symmetric objects -Boson Stars- that would be stopped from collapsing by Heisenberg's uncertainty principle. Because we assume a cold distribution of matter, all fields will be occupying the ground state, thus fixing the zero-momentum to be $p \sim \frac{1}{R} \sim m$, where m is the mass of the scalar field and R is the typical size of the object.

The study of cold Bose-stars was begun in the work by Ruffini and Bonazzola [1], and later revived by a number of authors [2,3,4]. It was found that, for free scalar fields, zero-node, non-singular and finite mass solutions to the combined Einstein and Klein-Gordon equations are possible and exhibit many different properties from the more familiar neutron stars, where gravitational collapse is avoided by Pauli's Exclusion Principle. For one thing , the condensation of bosons in the ground state produces an anisotropic stress that renders the concept of an equation of state useless [1]. Also, it was pointed out that the maximum mass for a boson star is of order $M_{crit} \sim M_{Planck}^2/m$, thus being much smaller than the Chandrasekhar mass $M_{Chan} \sim M_{Planck}^3/m^2$. Typically, for a scalar of mass $m \sim 10GeV$ we obtain $M_{max} \sim 10^{13}g$ which is 20 orders of magnitude lighter than the Sun.

If cold bosonic matter, in the form of axions [5], scalar neutrinos [6] or perhaps other as yet unknown scalar fields, is present in reasonable (by reasonable we mean at least as abundant as ordinary matter) quantities in the early Universe, it is not unplausible that condensation could occur followed by collapse to form mini black-holes that would contribute to the missing matter in the Universe. In fact, recent models for galaxy formation using cold dark matter and the inflationary scenario suggest that the ratio of baryonic (or luminous) matter to dark matter should be of order 10% .

The axion case is particularly interesting. In order to be consistent with energy loss rate of stars [7] and the cosmological mass density [5], the mass range for the axion should be between $1eV$ and $10^{-5}eV$. For the lowest limit we obtain Bose stars with masses of about 6 orders of magnitude smaller then the Sun, i.e., of the order of the Earth's mass. The possibility of condensation is enhanced by the fact that these fields are weakly interacting and slowly moving compared to the expansion rate of the Universe.

The above results are all related to the free field case. Recently Colpi, Shapiro and Wasserman [8] and Friedberg, Lee and Pang [9] have analysed the interacting case. In particular, the self-interacting quartic potential for a complex scalar field was shown to have rather different properties even for small coupling constant [8]. More important, for couplings of order unity, the critical mass is comparable to the Chandrasekhar limit. Due to the possible relevance of these objects to the early Universe, we consider it worthwhile to study other possible Lagrangians with non-trivial coupling terms. Accordingly, in this letter we will concentrate on finding star-like solutions to a complex scalar field with a non-minimal coupling to gravity. We will show that it is again possible to obtain physical solutions but that the critical masses are similar to the free case. Here we have to be careful because the non-minimal coupling, $\xi \mid \phi \mid^2 R$, generates an effective gravitational constant that, for a negative coupling constant ξ , will imply a critical central density where gravity becomes infinitely attractive and collapse unavoidable, as we will soon see.

The starting point for our calculation is the action

$$S = \int d^4x\sqrt{-g}\left[\left(\frac{1}{16\pi G} + \xi\phi^*\phi\right) R + g^{\mu\nu}\phi^*_{;\mu}\phi_{;\nu} - m^2\phi^*\phi\right] . \tag{1}$$

This action is invariant under a global phase transformation, $\phi \rightarrow e^{i\theta}\phi$, that implies in the conservation of its generator N , the total particle number. This conservation law is important since the binding energy for the spherical configuration is defined as $E_{bin} = M(\infty) - Nm$, where $M(\infty)$ is the mass of the star. For $E_{bin} \geq 0$ the system will disperse to infinity as N free particles.

By varying the action with respect to $g^{\mu\nu}$ and ϕ (or equivalently ϕ^*), we obtain the field equations,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi G T_{\mu\nu} \quad , \tag{2}$$

with

$$\begin{aligned} T_{\mu\nu} =& \phi^*_{,\mu}\phi_{,\nu} + \phi^*_{,\nu}\phi_{,\mu} - g_{\mu\nu}(g^{\alpha\beta}\phi^*_{,\alpha}\phi_{,\beta}) + g_{\mu\nu}m^2\phi^*\phi \\ &+ 2\xi\phi^*\phi(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R) - 2\xi g_{\mu\nu}g^{\alpha\beta}(\phi^*\phi)_{;\alpha\beta} + 2\xi(\phi^*\phi)_{;\mu\nu} \end{aligned} \tag{3}$$

and

$$g^{\alpha\beta}\phi_{;\alpha\beta} + (m^2 - \xi R)\phi = 0 \tag{4}$$

As we are assuming spherical symmetry, it is convenient to use the Schwarzschild coordinates

$$ds^2 = B(r)dt^2 - A(r)dr^2 - r^2 d\Omega^2 \quad . \tag{5}$$

For zero-node solutions, we can write the scalar field as (see ref.9)

$$\phi(r,t) = \Phi(r)e^{-i\omega t} \tag{6}$$

By using eqns. (5) and (6) into eqns. (2)-(4) we obtain

$$(1+2\xi\varphi^2)\left[\frac{B'}{rB}-\frac{A}{r^2}+\frac{1}{r^2}\right]=A\varphi^2\left(\frac{1}{B}-1\right)+\varphi'^2-2\xi\varphi\varphi'\left(\frac{B'}{B}+\frac{4}{r}\right) \tag{7.1}$$

$$\begin{aligned}(1+2\xi\varphi^2)\left[\frac{A'}{rA}+\frac{A}{r^2}-\frac{1}{r^2}\right]=&A\varphi^2\left(\frac{1}{B}+1\right)+\varphi'^2(1+4\xi)-2\xi\frac{B'}{B}\varphi'\varphi\\ &+4A\xi\varphi^2\left(1-\frac{1}{B}-\xi R\right)\end{aligned} \tag{7.2}$$

$$(1+2\xi\varphi^2+12\xi^2\varphi^2)R=(2+12\xi)\left(\frac{\varphi'^2}{A}-\frac{\varphi^2}{B}\right)+(4+12\xi)\varphi^2 \tag{7.3}$$

$$\varphi''+\frac{1}{2}\varphi'\left(\frac{B'}{B}-\frac{A'}{A}+\frac{4}{r}\right)-A\left(1-\frac{1}{B}-\xi R\right)\varphi=0 \quad , \tag{8}$$

where we have introduced $\varphi(r)=(8\pi G)^{1/2}\Phi(r)$, and the primes denote d/dx , $x\equiv mr$. The quantity $\frac{\omega^2}{m^2}$ has been absorbed in the definition of B . Eqn. (7.3) was obtained by taking the trace of eqn. (2) and by using the equation for the scalar field.

As usual, we can introduce the variable $\mathcal{M}(x)$ by

$$A(x)=[1-2\mathcal{M}(x)/x]^{-1} \quad . \tag{9}$$

The next step is to integrate numerically eqns. (7) and (8) subject to the boundary conditions $\mathcal{M}(0)=0$, $\varphi(0)=\varphi_0$, $\varphi'(0)=0$, and $\varphi(\infty)=0$. Also, φ was assumed to be nodeless. These standard conditions guarantee asymptotic flatness at infinity and avoid singularities at the origin. By integrating these equations, it is possible to obtain the total mass energy of the star, $\mathcal{M}(\infty)$. We follow the approach of ref. 8 and show, in Fig. 1, the variation of the Bose-star mass $\mathcal{M}(\infty)$ with the central density φ_0 for different values of the coupling $-0.4\le\xi\le0.6$. The result for $\xi=0$ agrees with previous results in the literature.

As with the self-interacting case [8], the mass of the star grows with the coupling strength $|\ \xi\ |$, for large values of $|\ \xi\ |$. By calculating the particle number N as a function of the central density for different configurations, we can show that the binding energy also grows with the coupling strength. In particular, it is a maximum at the maximum mass, as we show in Fig. 2. We note, however, that there is a curious change of behaviour for $\xi > 4.0$; the binding energy will then always be negative and the only sort of unstability is by gravitational collapse. Although beyond the maximum mass the star will presumably collapse (more about this later), it is interesting to try to understand the reason for this behaviour, even if at this point only heuristically. By looking at the scalar field equation (eqn. (8)), we can interpret the last term as the derivative of the potential that controls the dynamics of φ , with the second term being some sort of friction term. Notice that we can use eqn. (7.3) to write R in terms of φ and its derivatives. With this substitution we obtain roughly (i.e., ignoring some overall multiplicative term which is positive) a quartic potential for φ which, for positive ξ , has an attractive self-interacting coupling that favours the existence of a bound state. For negative ξ the interaction will be effectively repulsive favouring the dispersion of the star to infinity. Of course, with such primitive analysis, we have no simple explanation for the particular numbers obtained numerically.

In Fig. 3 we plot the maximum values of the star masses as a function of $\pm\xi$. The dots are the points obtained by the numerical integration while the continuous lines are the asymptotes

$$M_{max} \approx \ 0.73\xi^{1/2} M^2_{Planck}/m \quad , \xi \to +\infty$$

$$0.66 \mid \xi \mid^{1/2} M^2_{Planck}/m \quad , \xi \to -\infty \ . \tag{10.1}$$

Analogously, we obtain, for the maximum number of particles N_{max}

$$N_{max} \approx \ 0.88\xi^{1/2} M^2_{Planck}/m^2 \quad , \xi \to +\infty$$

$$0.72 \mid \xi \mid^{1/2} M^2_{Planck}/m^2 \quad , \xi \to -\infty \ . \tag{10.2}$$

In order to obtain the asymptotes for positive and negative ξ we simply have to do a rescaling in the field equations that will put the ξ dependence in the denominator and then compare the dominant powers at infinity. By writing $r \to \xi^{1/2} r$, $\varphi \to \frac{\varphi}{\xi^{1/2}}$ and remembering to take the absolute value of ξ in the negative limit, we obtain the two sets of asymptotic equations,

$$(1 \pm 2H)\left(\frac{B'}{rB} - \frac{A}{r^2} + \frac{1}{r^2}\right) = AH\left(\frac{1}{B} - 1\right) \mp \frac{B'H'}{B} \mp 4\frac{H'}{r} \tag{11.1}$$

$$(1 \pm 2H)\left(\frac{A'}{rA} + \frac{A}{r^2} - \frac{1}{r^2}\right) = AH\left(\frac{1}{B} + 1\right) \mp \frac{B'H'}{B} \pm \frac{1}{3}A\left(1 - \frac{1}{B}\right) - \frac{2}{3}AH \tag{11.2}$$

$$H'' = \frac{1}{6}\left(A(1 - \frac{1}{B}) \mp 2AH\right) + \frac{A'H'}{2A} - \frac{B'H'}{2B} - 2\frac{H'}{r} \tag{12}$$

where the top and bottom signs are for $\xi \to \pm\infty$, respectively, and we have introduced $H(r) = \varphi^2(r)$ for convenience.

The results of the numerical integration both for the total mass and for the particle number are shown in Fig. 4. Note that, as remarked before, the positive ξ limit has negative definite binding energy, while the negative ξ limit has a turnover point where the binding energy becomes positive. Another important characteristic of the negative ξ case is that, for any $\xi < 0$, there will be a critical value for the central density beyond which gravitational collapse is unavoidable; in fact, gravity becomes infinitely attractive at this critical point. From the action in eqn.(1) we can see that the effective gravitational constant is given by

$$\frac{1}{16\pi G_{eff}} = \frac{1}{16\pi G} + \xi\phi^2 \quad . \tag{13}$$

Thus, for negative ξ the integration stops at the critical value of ϕ for which $G_{eff} \rightarrow \infty$. The reader can verify this from Fig. 4. From explicit solutions it is possible to show that, whenever the central density approaches the critical density, $R \rightarrow \infty$ at the center of the star, showing the existence of a real singularity.

Contrary to the self-interacting case (ref. [8]), it is not possible to obtain an approximate equation of state for the large ξ limit. The discussion of gravitational equilibrium is more complicated here since we cannot apply the well-known stability theorems for fluid stars. Nevertheless, the remarkable qualitative similarity of the Bose star's mass dependence on the central density for the two models suggests that we can addopt their conclusions and conjecture that M_{max} represents the boundary between stable and unstable gravitational equilibrium.

Finally we remark that, for the axion case, the Peccei-Quinn symmetry is not exact (in which case $\xi = 0$) and, by QCD effects, one expects $\xi \sim (\Lambda_{QCD}/f_{axion})^2 \sim 10^{-20}$, for typical values of the axion decay constant [4]. The coupling strength is then very small and, as we can check from figures 1 or 3, the maximum mass is close to the free case $M_{max} \sim 10^{28}g$ for $M_{axion} \sim 10^{-5}eV$, a very light object, with a Schwarschild about $1cm$. Since the scalar field goes exponentially to zero with the radial coordinate, we can only have an estimate of the radius of the star, based in some arbitrary approximation. For example, we can take the radius to be where $2/3$ of the mass of the star lies or where the scalar field has relatively small values. Typically, the radius is one or two orders of magnitude bigger than the Schwarschild radius. If these objects were formed in the early Universe and collapse to become mini black-holes, we know from Hawking's theory that they should evaporate in a time scale of

$$\tau \sim 10^{10} yr \left(\frac{M}{10^{15} g} \right)^3 \quad . \tag{14}$$

Thus, for the axion star evaporation seems a remote possibility.

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Figure Captions

Figure 1. Bose-star mass in units of M^2_{Planck}/m as a function of the central density φ_0 for $\xi = -0.4,\ -0.2,\ 0.0,\ 0.2,\ 0.4,\ and\ 0.6$.

Figure 2. Bose-star mass in units of M^2_{Planck}/m (continuous line) and number of particles in units of M^2_{Planck}/m^2 (dotted line) for $\xi = 0.0,\ 3.0,\ and\ 8.0$.

Figure 3. A comparison between the maximum masses obtained numerically (points) and the asymptotic formulas ($for\ \xi \to \pm\infty$), eqn.(10).

Figure 4. Bose-star mass in units of $\mid \xi \mid^{1/2} M^2_{Planck}/m$ (continuous line) and number of particles in units of $\mid \xi \mid^{1/2} M^2_{Planck}/m^2$ (dotted line) for $\xi \to \pm\infty$.

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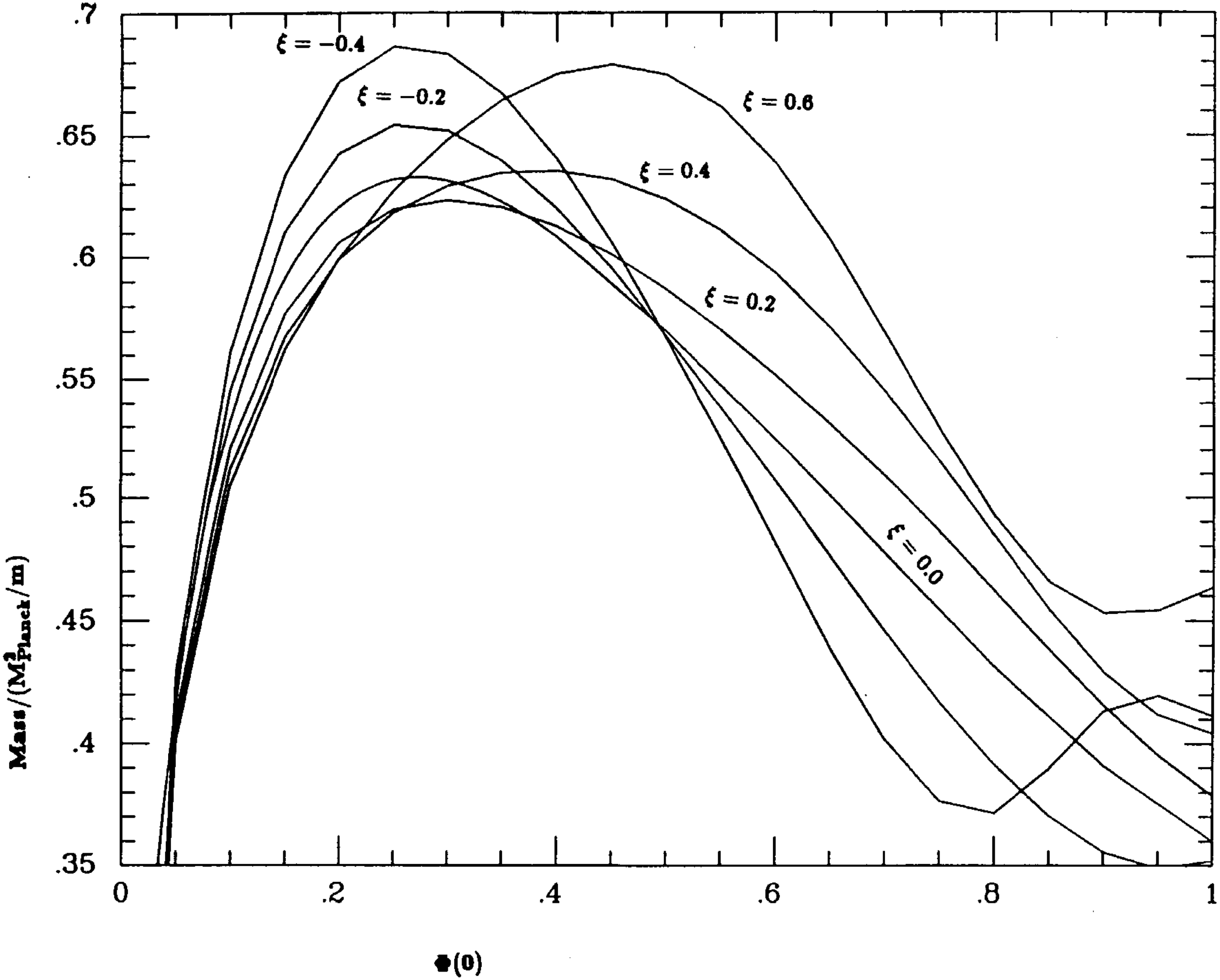
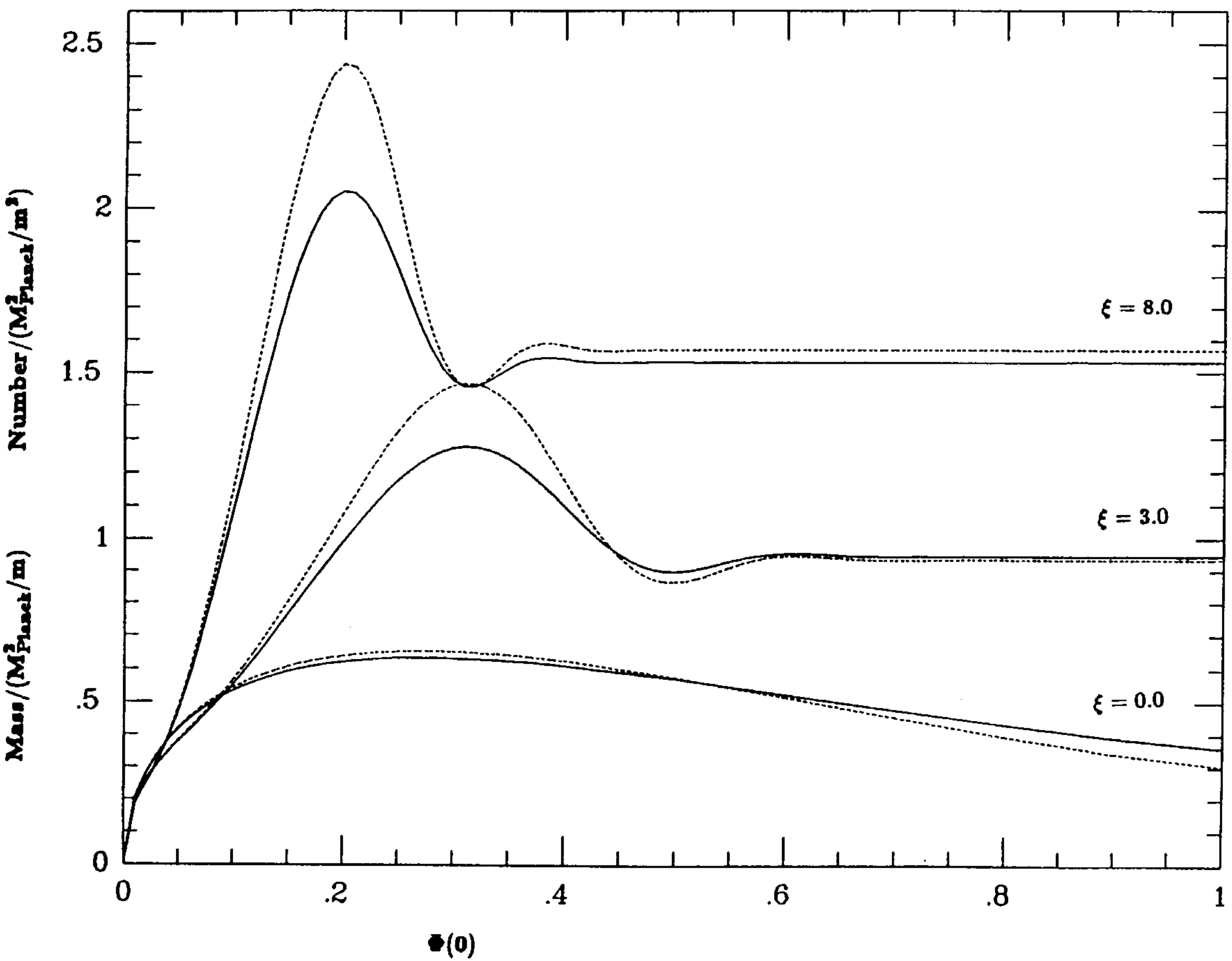
Fig. 1



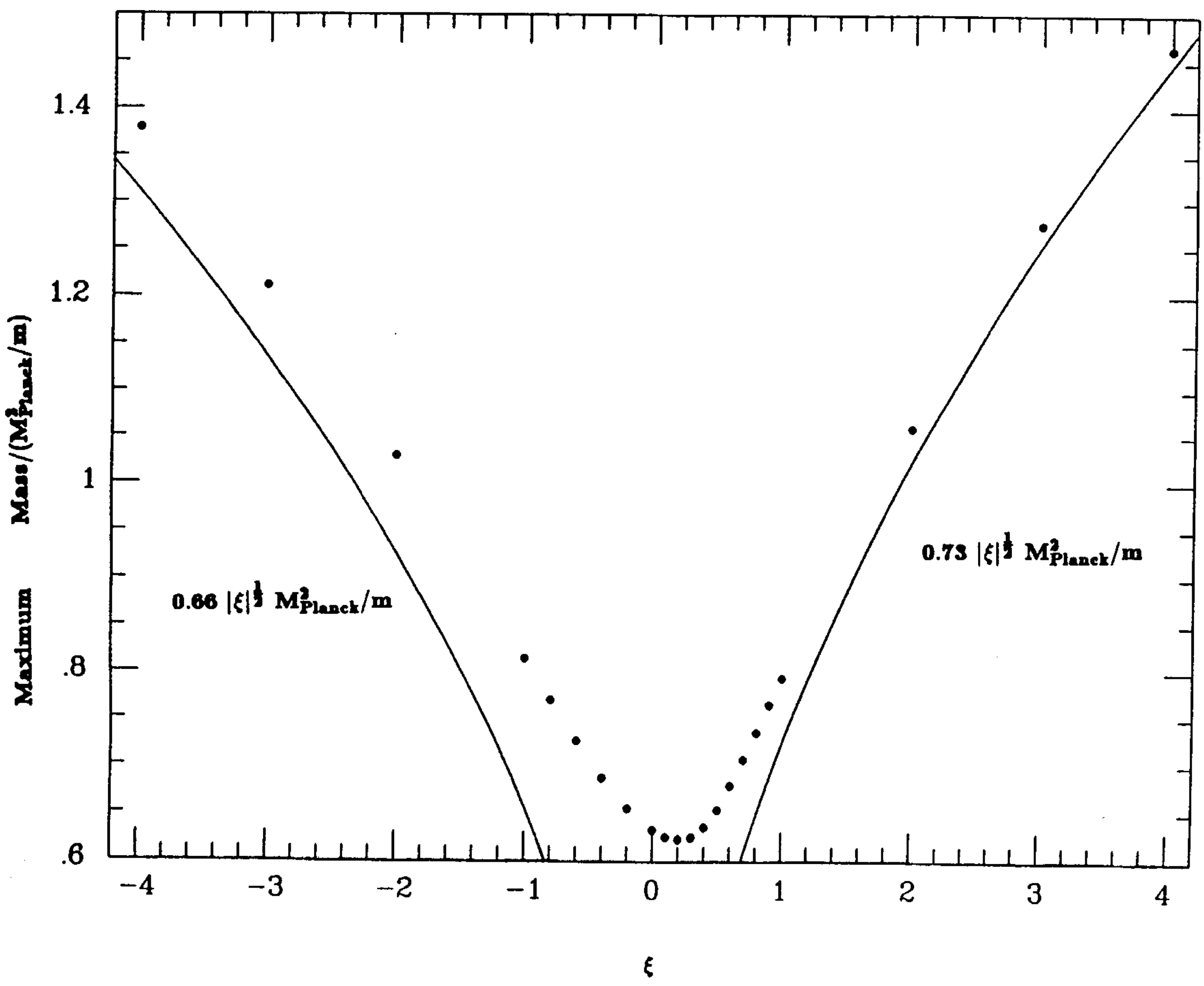
$\xi = -0.4$
$\xi = -0.2$
$\xi = 0.6$
$\xi = 0.4$
$\xi = 0.2$
$\xi = 0.0$
.7
.65
.6
.55
.5
.45
.4
.35
0
.2
.4
.6
.8
1
Mass/(M^2_{Planck}/m)
$\Phi(0)$

Fig. 2



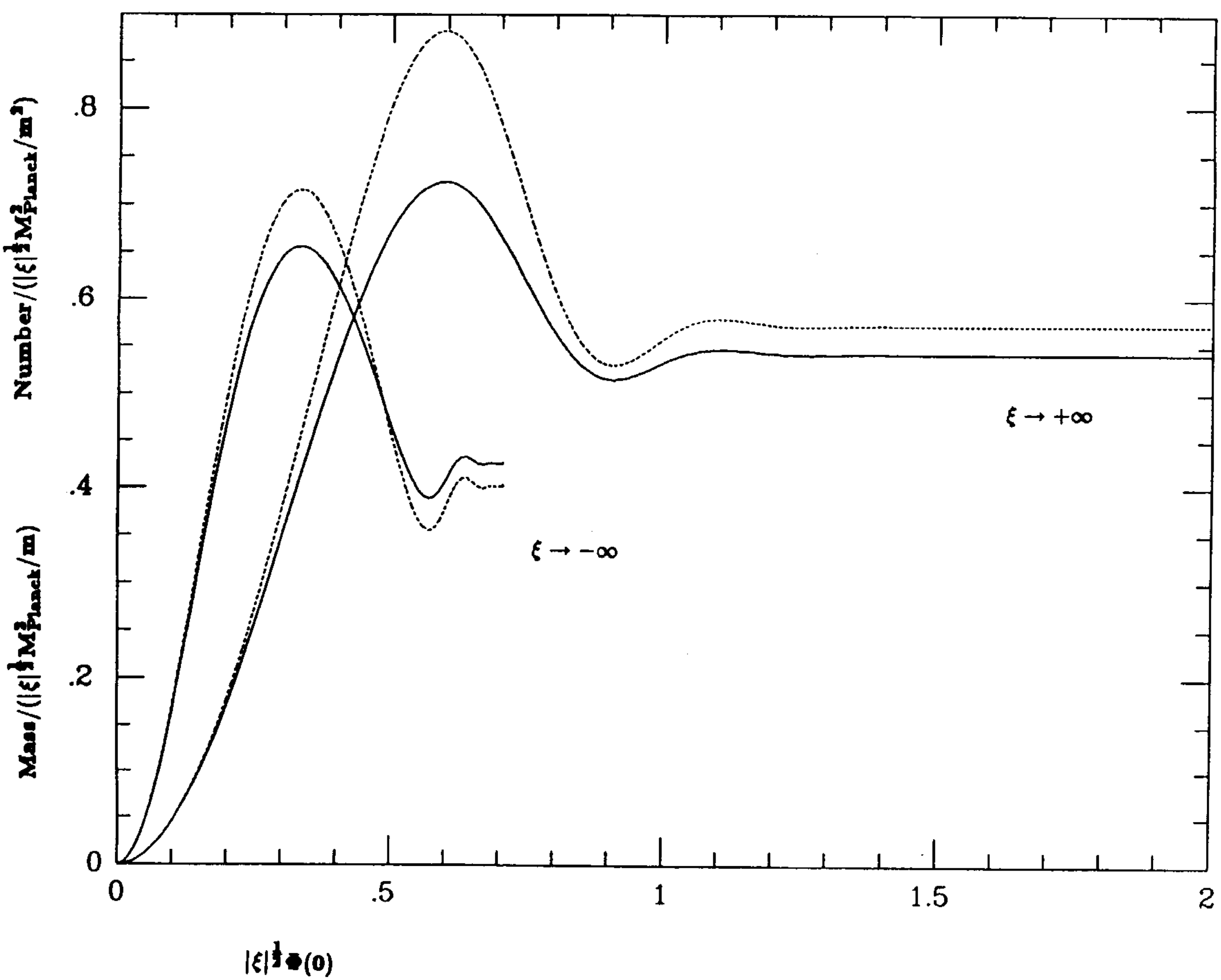
2.5
2
1.5
1
.5
0
Number/(M^2_{Planck}/m^2)
Mass/(M^2_{Planck}/m)
$\xi = 8.0$
$\xi = 3.0$
$\xi = 0.0$
0
.2
.4
.6
.8
1
$\Phi(0)$

Fig. 3



1.4
1.2
1
.8
.6
Maximum Mass/(M^2_{Planck}/m)
$0.66\ |\xi|^{\frac{1}{2}}\ M^2_{Planck}/m$
$0.73\ |\xi|^{\frac{1}{2}}\ M^2_{Planck}/m$
−4
−3
−2
−1
0
1
2
3
4
ξ

Fig. 4



Number/($|\xi|^{\frac{1}{2}}M^2_{Planck}/m^2$)
Mass/($|\xi|^{\frac{1}{2}}M^2_{Planck}/m$)
.8
.6
.4
.2
0
0
.5
1
1.5
2
$\xi \to -\infty$
$\xi \to +\infty$
$|\xi|^{\frac{1}{2}}\Phi(0)$